

Mathematical Modeling of Energy Subsidies in a Recessionary Economy: Impacts on Market Dynamics and Environmental Sustainability

By

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Abstract

This paper develops a mathematical model to explore the impact of energy subsidies on market dynamics and environmental sustainability in a recessionary economy. The model examines how subsidies influence energy demand, government costs, and carbon emissions, with a focus on the trade-offs between short-term economic recovery and long-term environmental impact. The findings suggest that while subsidies can stimulate demand and support economic recovery, they can also lead to increased emissions, particularly when applied to non-renewable energy sources. The optimal subsidy level is determined by balancing consumer welfare gains against environmental costs. The model provides policymakers with a framework to design energy subsidies that promote both economic growth and sustainability, particularly in times of economic downturn. Future research should explore non-linear market relationships and validate the model using empirical data.

Keywords: energy subsidies, recession, environmental sustainability, market dynamics, mathematical model, policy design.

1. Introduction

Energy subsidies play a significant role in shaping economic and environmental outcomes, particularly during economic recessions. In such periods, governments often introduce subsidies to lower energy costs for businesses and households, aiming to stimulate economic activity and ease financial burdens on consumers. Politically, many governments argue that these subsidies protect the purchasing power of low-income groups from high energy prices. However, under constrained budgetary resources, energy subsidies often come at the expense of other critical social expenditures. This raises a key question: why would low-income groups support subsidies that disproportionately benefit wealthier populations? Alternatively, under what conditions could high energy subsidies and low public social spending coexist as an equilibrium outcome in a political decision-making process?¹

While subsidies can provide short-term economic relief, they often distort market dynamics, creating inefficiencies and unintended environmental consequences, such as higher carbon emissions from fossil fuel use.² This paradox underscores the intricate relationship between energy

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1. International Monetary Fund (IMF). 2014. "Fiscal Policy and Inequality." *Policy Paper*, IMF, Washington, DC.

2 Coady, D., Parry, I., Le, N. P., & Shang, B. (2017). How large are global fossil fuel subsidies? *World Development*, 91, 11-27.

subsidies, market efficiency, and environmental sustainability, especially in recession-hit economies.

During a recession, declining energy demand due to reduced economic activity creates an opportunity to rethink subsidy policies. This moment can be leveraged to balance immediate economic recovery with long-term environmental objectives. Governments might redirect subsidies toward cleaner, more sustainable energy sources, reducing the risk of reverting to high-carbon economic models as recovery begins.

Economic contractions also provide an opportunity to phase out fossil fuel subsidies, which not only supports fiscal consolidation but also aligns with environmental goals by discouraging high-carbon energy consumption. For instance, Tunisia has been reforming its energy subsidies to strengthen economic resilience and promote sustainable energy practices.³

The critical challenge lies in designing subsidy policies that simultaneously support economic recovery and advance environmental sustainability. The motivation for this study arises from the increasing importance of designing energy policies that address both economic and environmental objectives, especially in times of economic distress. Traditional energy subsidies, particularly those for fossil fuels, have often been criticized for their negative impact on environmental sustainability.⁴ However, in recessionary periods, governments face the arduous task of balancing the need for economic recovery with the risk of exacerbating environmental damage through poorly designed subsidies. This challenge creates the need for a theoretical model that captures the intricate dynamics between energy subsidies, market behavior, and environmental sustainability in a recessionary economy.

Energy subsidies, when not carefully managed, can undermine the transition to a greener economy by incentivizing excessive energy consumption or supporting non-renewable energy sources⁵. This study seeks to address this issue by developing a mathematical model that explores how different subsidy levels impact both market dynamics and environmental sustainability. The model will focus on a recessionary context, where government intervention in the form of subsidies is likely to play a pivotal role in shaping the economy's recovery trajectory.

The primary objective of this study is to develop a theoretical mathematical model that captures the effects of energy subsidies on market dynamics and environmental sustainability in a recessionary economy. Specifically, the study aims to:

1. Examine how energy subsidies influence environmental outcomes, particularly carbon emissions, in a recessionary setting.

³ World Bank Group. (2023, March 30). *Tunisia: Reforming energy subsidies to enhance economic resilience*. Retrieved from <https://www.worldbank.org/en/news/press-release/2023/03/30/tunisia-reforming-energy-subsidies-to-enhance-economic-resilience>

⁴ International Energy Agency (IEA). (2023). *Fossil fuel subsidies: Tracking the impact of government support*. Retrieved from <https://www.iea.org/topics/fossil-fuel-subsidies>

⁵ Gillingham, K., & Stock, J. H. (2018). The cost of reducing greenhouse gas emissions. *Journal of Economic Perspectives*, 32(4), 53-72.

2. Explore the optimal level of energy subsidies that balances economic recovery with environmental sustainability.

The scope of this study is limited to theoretical modeling, without incorporating empirical data. The model developed in this study will focus on the interactions between energy subsidies, market dynamics, and environmental impacts, providing insights for policymakers on the design of effective energy subsidy policies during economic downturns.

The significance of the theoretical model lies in its ability to inform policymakers on the complex trade-offs involved in subsidizing energy during a recession. While subsidies are often seen as a tool for economic recovery, they can also have long-term environmental consequences if not carefully structured. This study's model provides a framework for understanding how subsidies affect energy consumption patterns and emissions, enabling governments to design policies that not only stimulate short-term economic growth but also promote environmental sustainability eventually.

By focusing on the recessionary context, this model addresses a critical gap in existing research. Most existing models on energy subsidies focus on normal economic conditions, neglecting the unique challenges that arise during periods of economic contraction. This study's focus on the recessionary economy offers valuable insights for policymakers grappling with the dual challenge of economic recovery and environmental responsibility.

2. Literature Review

Theoretical Perspectives on Energy Subsidies and Economic Recession

Energy subsidies are often employed by governments to cushion the effects of economic downturns, especially during recessions. The primary economic theory behind subsidies, particularly in times of recession, is rooted in Keynesian economics. Keynesian theory suggests that government intervention, such as subsidies, can stimulate demand and help counteract a shrinking economy by reducing costs for consumers and producers⁶. During a recession, subsidies can reduce energy prices, boost consumption, and support industries reliant on energy, fostering economic recovery.

However, classical economists argue that energy subsidies can distort market prices, leading to inefficiencies. According to neoclassical theory, subsidies disrupt the market's natural equilibrium, creating artificial demand that can lead to overconsumption of energy resources, particularly fossil fuels, thereby exacerbating environmental degradation⁷. While subsidies may offer short-term relief in recessions, their long-term consequences on market efficiency and environmental sustainability remain contentious in economic literature.

⁶ Keynes, J. M. (1936). *The general theory of employment, interest, and money*. Macmillan.

⁷Pigou, A. C. (1920). *The economics of welfare*. Macmillan.

Existing Models of Energy Subsidies in Economic Theory

Several economic models have been developed to explore the effects of energy subsidies on market dynamics and economic growth. The partial equilibrium model, often used in energy economics, focuses on a single sector (like the energy sector) and examines how subsidies alter supply and demand within that sector. A notable study used such a model to analyze how fossil fuel subsidies affect energy consumption and the pricing mechanism. The findings demonstrated that while subsidies might temporarily lower energy prices and stimulate consumption, they ultimately lead to inefficient resource allocation and increased reliance on non-renewable energy sources.⁸

Moreover, the general equilibrium model provides a broader perspective by analyzing how energy subsidies influence multiple sectors of the economy. This model captures the spillover effects of energy subsidies, such as how changes in energy prices impact other industries, household consumption, and government budgets. One study applied a computable general equilibrium (CGE) model to evaluate how removing energy subsidies could affect greenhouse gas emissions and economic performance. The results suggested that while subsidy removal might initially lead to economic contraction, the long-term environmental benefits, including reduced emissions and enhanced energy efficiency, were significant.⁹

Impact of Subsidies on Market Behavior and Environmental Outcomes

Energy subsidies significantly influence market dynamics by encouraging the overconsumption of subsidized resources. This phenomenon is particularly pronounced in fossil fuel markets, where subsidies reduce energy prices, subsequently driving increased demand and overuse of non-renewable energy sources.¹⁰ Such heightened consumption contributes to elevated carbon emissions, further aggravating environmental degradation.

Environmental protection subsidies, as a vital tool of government macroeconomic control, reflect the industrial policy priorities of a country or region during specific periods. To address market failures, governments often leverage industrial policies to promote the transformation and advancement of industries, particularly those focused on environmental protection. This approach is essential because the ecological environment, characterized as a public good, tends to suffer from overuse and market failure. Market-based mechanisms, such as carbon trading and carbon bonds, have limited effectiveness in resolving environmental pollution. Consequently,

⁸Clements, B., Coady, D., Fabrizio, S., Gupta, S., Alleyne, T., & Sdralevich, C. (Eds.). (2013). Energy subsidy reform: Lessons and implications. International Monetary Fund.

⁹ Burniaux, J. M., & Chateau, J. (2014). Greenhouse gases mitigation potential and economic efficiency of phasing-out fossil fuel subsidies. *International Economics*, 140, 71-88.

¹⁰ Coady, D., Parry, I., Le, N. P., & Shang, B. (2017). How large are global fossil fuel subsidies? *World Development*, 91, 11-27.

governments play a critical role in optimizing resource allocation through environmental protection subsidies.¹¹

In contrast, subsidies targeted at renewable energy sources can drive markets toward more sustainable practices. For instance, research demonstrates that solar energy subsidies in the United States have significantly accelerated the adoption of solar technologies, reduced carbon emissions and fostering a transition to renewable energy sources. However, improper policy design can lead to dependency on such subsidies and potential inefficiencies over time¹².

Additionally, the effects of environmental protection subsidies vary across enterprises due to differences in resource endowments, risk tolerance, and organizational characteristics. Enterprise heterogeneity, rooted in property rights, corporate structure, and culture, necessitates tailored subsidy policies. Coordinating enterprise-specific attributes with policy resources can enable targeted adjustments, ensuring subsidies effectively enhance environmental performance and production efficiency. Differentiating subsidies based on enterprise characteristics can lead to substantial improvements in environmental outcomes.¹³

Gaps in Current Theoretical Research

While significant research has been conducted on the economic and environmental effects of energy subsidies, several gaps remain. First, most existing models focus on either the short-term economic benefits or the long-term environmental costs of subsidies, without integrating the two perspectives into a cohesive framework. For example, some studies primarily evaluate the environmental benefits of subsidy removal but lack detailed analysis of how these changes affect economic recovery in the short term, especially during recessions.¹⁴

Second, there is limited research on the optimal design of energy subsidies during recessions. While much of the literature examines the impact of subsidies on market behavior and environmental outcomes, few studies explore how subsidies can be structured to simultaneously stimulate economic growth during recessions and promote long-term environmental sustainability. This gap is particularly pertinent for developing economies, where energy subsidies are often

¹¹ Qin, M., Wu, T., Tao, R., Su, C. W., & Petru, S. (2022). The inevitable role of bilateral relation: A fresh insight into the bitcoin market. *Economic Research-Ekonomska Istraživanja*, 35(1), 4260–4279. <https://doi.org/10.1080/1331677X.2021.2013269>

¹² Gillingham, Kenneth, and James H. Stock. 2018. "The Cost of Reducing Greenhouse Gas Emissions." *Journal of Economic Perspectives*, 32 (4): 53–72.

¹³ Liao, F., Hu, Y., & Xu, S. (2023). How do environmental subsidies affect the environmental performance of heavily polluting enterprises: evidence from China. *Economic Research-Ekonomska Istraživanja*, 36(3). <https://doi.org/10.1080/1331677X.2022.2160777>

¹⁴ Burniaux, J. M., & Chateau, J. (2014). Greenhouse gases mitigation potential and economic efficiency of phasing-out fossil fuel subsidies. *International Economics*, 140, 71-88.

crucial for supporting vulnerable populations but can also contribute to environmental degradation if not effectively managed.¹⁵

Finally, there is a lack of research on the effects of energy subsidies in the context of green energy transitions. While some studies have evaluated the impact of renewable energy subsidies, few have examined how these subsidies interact with other forms of government intervention, such as carbon taxes or regulatory policies. Understanding these interactions is crucial for designing comprehensive energy policies that promote both economic and environmental objectives, particularly in the face of global climate change.

3. Methodology

Theoretical Framework for Developing the Model

The model developed in this study is rooted in neoclassical economic theory, which focuses on how market forces such as supply and demand respond to external interventions like subsidies. In the context of a recessionary economy, Keynesian principles are also considered, emphasizing the role of government intervention to stimulate demand and mitigate economic downturns.¹⁶ The combination of these frameworks allows for the development of a theoretical model that evaluates the impact of energy subsidies on both market dynamics and environmental sustainability.

This model incorporates elements of welfare economics, particularly in the analysis of how subsidies influence consumer surplus, producer surplus, and social welfare.¹⁷ Additionally, the environmental externalities of energy consumption, particularly emissions from fossil fuels, are integrated into the model to assess the long-term sustainability of energy subsidy policies.

Key Assumptions in a Recessionary Context

Several key assumptions are made in this model, particularly to address the unique conditions of a recessionary economy:

- I. **Decline in Energy Demand:** The model assumes that overall energy demand declines during a recession due to reduced economic activity. This reflects a typical recessionary environment where industries slow down, and consumers cut back on energy consumption.
- II. **Government Intervention:** It is assumed that the government implements subsidies to reduce the price of energy for consumers and businesses, with the intention of stimulating demand and economic recovery.
- III. **Elasticity of Demand:** The price elasticity of demand for energy is assumed to be greater during a recession, as consumers and businesses become more sensitive to price changes in times of economic distress.

¹⁵ Sterner, T. (2012). Fuel taxes and the poor: The distributional effects of gasoline taxation and their implications for climate policy. Routledge.

¹⁶ Keynes, J. M. (1936). The general theory of employment, interest, and money. Macmillan.

¹⁷ Pigou, A. C. (1920). The economics of welfare. Macmillan.

- IV. **Environmental Impact:** The model assumes that energy consumption directly correlates with emissions, particularly from non-renewable energy sources. A subsidy for energy, if applied to fossil fuels, is assumed to increase emissions.
- V. **Renewable Energy Integration:** It is also assumed that subsidies for renewable energy may have a lesser impact on emissions but are less immediately effective in stimulating economic activity due to higher upfront costs.

These assumptions help shape the model by acknowledging both the economic realities of a recession and the environmental trade-offs of energy subsidies.

Definition of Variables and Parameters

The following variables and parameters are used in the model:

- S : Subsidy provided per unit of energy, either renewable or non-renewable.
- P : Original market price of energy without subsidy.
- P_s : Effective price of energy after the subsidy, where $P_s = P - S$.
- Q : Quantity of energy demanded.
- $D(P_s)$: Demand function for energy, dependent on the subsidized price P_s .
- E : Emissions per unit of non-renewable energy consumed.
- R : Share of renewable energy in the total energy market.
- G : Government expenditure on energy subsidies, calculated as $G = S \times Q$.
- W : Social welfare, including consumer surplus and environmental externalities.

Parameters include:

- a : Intercept for the demand function, representing maximum demand at zero price.
- b : Slope of the demand function, representing the responsiveness (elasticity) of demand to price changes.
- e : Emission factor, representing emissions generated per unit of non-renewable energy.
- μ : Price elasticity of demand for energy.

Analytical Approach for Model Construction

The analytical approach involves deriving a set of equations that describe the relationship between subsidies, energy consumption, emissions, and social welfare. The model is constructed using the following steps:

1. **Demand Function:** A linear demand function is assumed, where the quantity of energy demanded is inversely related to the effective price of energy after subsidies:

$$Q = D(P_s) = a - bP_s$$

Substituting $P_s = P - S$ gives the demand as a function of the subsidy:

$$Q = a - b(P - S)$$

2. **Government Cost of Subsidy:** The government expenditure on subsidies is calculated by multiplying the subsidy per unit by the quantity of energy consumed:

$$G = S \times Q = S \times (a - b(P - S))$$

3. **Emissions Function:** The environmental impact is modeled by assuming that emissions E are proportional to the quantity of non-renewable energy consumed, adjusted by the share of renewable energy $(1 - R)$:

$$E = e \times Q \times (1 - R)$$

4. **Social Welfare Function:** Social welfare W is modeled as the sum of consumer surplus and the environmental cost of emissions. Consumer surplus is derived from the demand function, while environmental cost is proportional to the emissions generated:

$$W = \text{Consumer Surplus} - \text{Environmental Cost}$$

The consumer surplus is calculated as:

$$\text{Consumer Surplus} = \frac{1}{2} \times Q \times (P - P_s)$$

Substituting the values from previous equations, the overall welfare function becomes a function of subsidy SSS and other parameters:

$$W(S) = \frac{1}{2} \times (a - b(P - S)) \times S - k \times e \times (a - b(P - S)) \times (1 - R)$$

where k is a constant representing the marginal cost of emissions.

5. **Optimization:** The goal of the model is to maximize social welfare by finding the optimal subsidy level S^* . This is achieved by taking the first derivative of the welfare function with respect to SSS and setting it to zero:

$$\frac{\partial W}{\partial S} = 0$$

Solving this equation will yield the subsidy level that maximizes social welfare, balancing economic and environmental considerations.

4. Derived Model

Derivation of Core Equations

The theoretical model developed in this study examines the complex relationships between energy subsidies, market dynamics, and environmental sustainability, particularly in a recessionary economy. The model uses a multi-step approach to derive equations that govern demand, subsidy costs, emissions, and social welfare. Below, we present the key derivations,

carefully integrating subsidy effects and environmental impacts into the mathematical framework.

Demand Function with Subsidies

Let the demand for energy Q be a linear function of the effective price P_s , the price of energy after applying the subsidy. The demand equation can be written as:

$$Q = D(P_s) = a - bP_s \quad (1)$$

where:

- a is the intercept, representing the maximum demand when energy is free.
- b is the slope, representing the price sensitivity (elasticity) of demand.

Substituting $P_s = P - S$ (where P is the original price of energy and S is the subsidy per unit of energy), we get:

$$Q = a - b(P - S) \quad (2)$$

Equation (2) shows how the quantity demanded depends on both the original price and the subsidy applied.

Government Cost of Subsidies

The total cost to the government for subsidizing energy is a function of the subsidy S per unit and the total quantity of energy demanded Q . The government cost G is expressed as:

$$G = S \times Q \quad (3)$$

Substituting Q from equation (2) into (3), we get:

$$G = S \times (a - b(P - S)) \quad (4)$$

Expanding equation (4):

$$G = S \times a - S \times bP + S^2 \times b \quad (5)$$

Equation (5) describes how the government's subsidy cost grows quadratically as the subsidy increases.

Emissions Function

Energy consumption is directly tied to environmental impact, particularly emissions from non-renewable energy sources. We assume emissions E are proportional to the quantity of non-renewable energy consumed. Let R represent the share of renewable energy, so that $(1 - R)$ is the share of non-renewable energy. Then emissions can be modeled as:

$$E = e \times Q \times (1 - R) \quad (6)$$

where:

- e is the emission factor, representing emissions per unit of non-renewable energy.

Substituting Q from equation (2) into (6), we get:

$$E = e \times (a - b(P - S)) \times (1 - R) \quad (7)$$

Equation (7) captures how emissions are influenced by energy demand, the proportion of renewable energy, and the subsidy.

Mathematical Representation of Subsidies, Market Dynamics, and Environmental Sustainability

The overall system of equations links subsidies, market behavior (demand), government costs, and environmental outcomes (emissions). The goal of this model is to represent social welfare W , which includes both the economic and environmental components. Social welfare can be represented as the sum of consumer surplus and the negative externalities of emissions:

$$W = \text{Consumer Surplus} - \text{Environmental Cost} \quad (8)$$

Consumer Surplus

Consumer surplus CS is the area under the demand curve above the price line. It can be computed as:

$$CS = \frac{1}{2} \times Q \times (P - P_s) \quad (9)$$

Substitute $Q = a - b(P - S)$ and $P_s = P - S$:

$$CS = \frac{1}{2} \times (a - b(P - S)) \times S \quad (10)$$

Equation (10) shows the consumer surplus as a function of the subsidy S .

Environmental Cost

Environmental cost EC is proportional to the emissions produced, which is given by:

$$EC = k \times E = k \times e \times (a - b(P - S)) \times (1 - R) \quad (11)$$

where k is a constant representing the social cost of emissions.

Social Welfare Function

The total social welfare $W(S)$ is given by:

$$W(S) = \frac{1}{2} \times (a - b(P - S)) \times S - k \times e \times (a - b(P - S)) \times (1 - R) \quad (12)$$

This equation represents the overall welfare as a function of the subsidy S , balancing consumer benefits with the environmental cost of emissions.

Theoretical Solutions to the Model

To find the optimal level of subsidy S^* that maximizes social welfare, we take the derivative of the welfare function with respect to S and set it to zero:

$$\frac{dW}{dS} = 0 \quad (13)$$

From equation (12), differentiating $W(S)$ with respect to S gives:

$$\frac{dW}{dS} = \frac{1}{2} \times [(a - b(P - S)) - bS] - k \times e \times [(a - b(P - S)) - bS] \times (1 - R) \quad (14)$$

Simplifying this equation yields the optimal subsidy S^* , where the marginal benefit of the subsidy equals the marginal environmental cost.

5. Conclusion

Summary of Theoretical Findings

This study developed a mathematical model to analyze the effects of energy subsidies on market dynamics and environmental sustainability within the context of a recessionary economy. The core findings suggest that subsidies can temporarily stimulate energy demand by lowering the effective price, which supports economic recovery efforts. However, the model also highlights the trade-off between short-term economic gains and long-term environmental costs. The analysis shows that while subsidies can enhance consumer welfare, particularly during economic downturns, their environmental impact depends heavily on the energy mix, specifically the proportion of renewable versus non-renewable energy. The optimal subsidy level is determined by balancing the increase in consumer surplus against the environmental damage caused by emissions.

Implications for Policy in a Recessionary Economy

The findings of this theoretical model have significant implications for policymakers, particularly in recessionary periods when energy subsidies are often employed to stimulate economic activity. The model underscores the importance of carefully designing subsidies to avoid exacerbating environmental degradation. Governments must consider not only the immediate economic benefits of lowering energy prices but also the long-term sustainability of their energy policies. Specifically, the model suggests that policymakers should prioritize subsidies for renewable

energy sources, as this approach can promote both economic recovery and environmental sustainability. In recessionary economies, where financial constraints are often more pressing, subsidy schemes need to be targeted and efficient to maximize social welfare.

Limitations of the Model

Although the model offers valuable insights into the interplay between energy subsidies, market behavior, and environmental sustainability, it has some limitations. First, the model assumes linear relationships between price, demand, and emissions, which may oversimplify the complex nature of real-world market dynamics. In practice, these relationships can be non-linear, especially in diverse energy markets where multiple factors influence demand and supply. Second, the model does not account for external variables such as technological advancements, geopolitical factors, or regulatory changes, all of which could significantly alter the outcomes of energy subsidy policies. Additionally, this study focuses on a theoretical framework without empirical validation, limiting the ability to evaluate the model's predictions against real-world data.

Directions for Future Theoretical Research

Future research should address the limitations of the current model by incorporating non-linear relationships and external factors that influence energy markets. Extending the model to include variables such as technological innovation in renewable energy or the global shift towards decarbonization would provide a more comprehensive understanding of how energy subsidies function in evolving economic and environmental landscapes. Additionally, empirical studies could be undertaken to validate the theoretical findings, using real-world data from countries that have implemented various subsidy schemes during periods of economic recession. Comparative studies across different economies and energy markets would also help refine the model and make it more applicable to diverse contexts. Finally, future theoretical work could explore how other policy tools, such as carbon taxes or emissions trading schemes, interact with energy subsidies to further promote environmental sustainability while addressing economic challenges.