

A Simplified Model to Perform Nodal Analysis for Oil Wells: A Novel Prediction Method

By

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ABSTRACT

Existing models for performing Nodal Analysis for a production system without displaying charts, has been identified to be somewhat complex and can only be solved by empirical methods. However, in this study, a simplified sub-model (Rotimi and Wilfred model) was developed from an existing complex model (Guo-Ghalambor). In the course of simplifying the complex Guo-Ghalambor model, it was first solved analytically using differential calculus to obtain a sub-model so that an iteration method such as Newton Raphson coupled with advanced computer programming used to arrive at a definite solution. With the use of a computer model, NODARES, nodal analysis was conducted for a range of data points. The results obtained from the tool revealed that, convergence is fast on the part of using the Rotimi and Wilfred model to perform nodal analysis. The developed computer model was validated using a range of data points in the determination of the operating pressure and flow rate at which the crude oil will be produced. Also, a comparative analysis between Excel Solver and the developed computer model proved that NODARES is most suitable to perform nodal analysis without displaying charts. Moreover, for visual representation, Poettmann Carpenter model was also used to perform nodal analysis.

Keywords: Nodal analysis, Rotimi and Wilfred model, NODARES tool, computer model, simplified model.

INTRODUCTION

In the early life of a field, after geologic and engineering information have proved the presence of economically viable hydrocarbons; it will be necessary to drill a development well. A development well is usually drilled taking into consideration, present and possible future occurrences. The producer well is usually designed so as to maintain an optimum production rate. Conventionally, the well deliverability is defined by the flow of reservoir fluid from the reservoir into the wellbore. This flow can be analyzed in a plot with pressure as the ordinate and flow rate as abscissa. This relationship is called Inflow Performance Relationship (IPR). For crude oil to be produced to the surface, a tubing is needed. Thus, the tubing performance relationship must be evaluated. This is also called Vertical Lift Performance (VLP). Nodal Analysis is therefore performed by combining these two techniques (IPR and VLP) in the prediction of the operating flow rate and pressure.

The well deliverability is hitherto defined by the intersection of the two performance curves as shown in Figure 1 below. Ghalambor et al.¹ suggested that a knowledge of the expected flow rate will enable the top management of the oil industry to determine whether it would be necessary to change the tubing size or other parameters, so as to specify a particular flow rate.

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¹ Ghalambor, A., Lyons, W. C. & Guo, B. (2007). *Petroleum Production Engineering*. Elsevier Science and Technology Books.

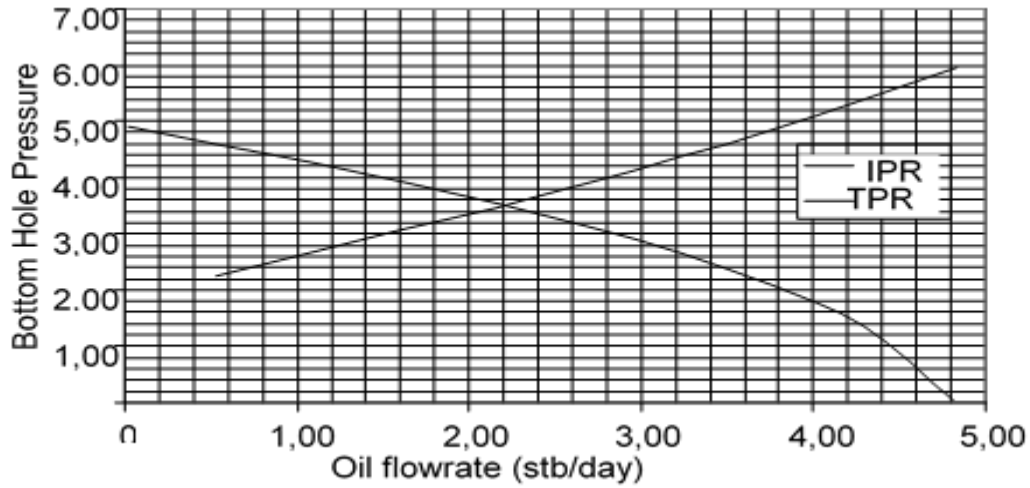


Figure 1: Typical chart illustrating Nodal Analysis²

The present day oil industry makes use of company developed computer models to perform certain operations. Nodal Analysis can be performed with ease by using the suitable computer models. Unfortunately, acquiring licence to use most computer models by some indigenous companies and oil operators is quite expensive. Therefore, it is necessary to develop a software locally that would be cheap and easily accessible with little constraint. Moreover, existing models for performing Nodal Analysis without displaying charts, has been identified to be somewhat complex and can only be solved by empirical methods.

This study is therefore aimed at developing an analytic computer model that will perform nodal analysis for a production system. The study provides a simplified solution to the complex Guo-Ghalambor empirical by using Newton Raphson iteration method thus, providing a feasible solution to the Guo-Ghalambor model which can be used to predict the bottomhole flowing pressure and its corresponding flowrate without plotting graphs. The computer model developed can be used to perform complex calculations in the shortest possible time.

STATEMENT OF THEORY AND DEFINITIONS

A number of existing models which can be used to perform nodal analysis and reservoir volume estimation are presented in the sub-sections below.

Inflow Performance Relationship Model

Production engineers use IPR to evaluate reservoir deliverability. The IPR curve is a graph showing the relationship between the flowing bottom-hole pressure and liquid production rate. A typical straight line IPR curve is shown in Figure 2. The reciprocal of the slope of the IPR curve, is the productivity index which is measured in stb/day-psi. Mathematically, it is evaluated as;

$$J = \frac{q}{P_e - P_{wf}} \quad (1)$$

Where J is the productivity index. It is imperative to note that J is not a constant in the two-phase flow region.

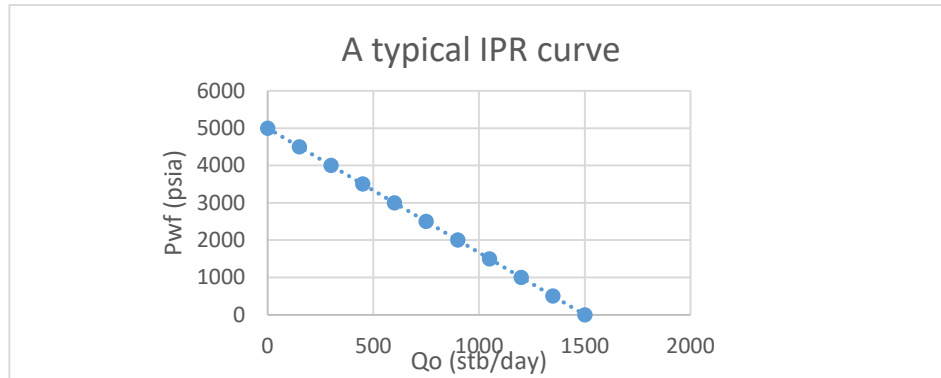


Figure 2: Typical IPR curve

More often than not, Well IPR curves are constructed using reservoir inflow models, which can either be from a theoretical basis or an empirical basis. It is imperative however, to validate these models with test points in field applications.

IPR for Single (Liquid)-Phase Reservoirs

Single phase IPR assumes that the reservoir fluid is purely liquid (i.e. single phase). This assumption is only valid for undersaturated oil reservoirs – where the pressure condition of the wellbore bottom, is higher than the bubble point pressure of the reservoir fluid. The equation that describes the productivity index for a flowing bottom-hole pressures above the bubble-point pressure is as shown in eq. (1).

At the point on the IPR curve where the bottom hole flowing pressure is zero, the absolute open flow (AOF) is defined by the product of productivity index (J) and the reservoir pressure².

IPR for Double-Phase Reservoirs

At a bottom hole flowing pressure less than the bubble bubble-point pressure of the reservoir fluid, straight line IPR model cannot be competent. This is because, below the bubble point pressure of a fluid, solution gas evolves from the oil and form free gas. The free gas affects the flow regime of the fluid within the reservoir by making the flow to be somewhat slow. Also, the evolvment of gas causes the oil to shrink due to the increase of oil viscosity. The combination of the flow regime effect and the viscosity effect results in lower oil production rate at a given bottom-hole pressure. This makes the IPR curve to deviate from the linear trend at a pressure below bubble-point pressure. Higher pressure values causes lower deviation from the linear trend and vice versa. Empirical equations are required for the modelling of two-phase IPR. These empirical equations consist of

² Guo, B., & Ghalambor, A. (2002). *Gas Volume Requirements for Underbalanced Drilling Deviated Holes*. Tulsa: PennWell Corporation.

Vogel's³ equation extended by Standing⁴, the Fetkovich⁵ equation, Bandakhlia and Aziz's⁶ equation, Economides equation and Retnanto⁷. Vogel's equation is still the most used empirical equation in the industry. It is written as:

$$Q = q_{max} * \left(1 - 0.2 * \left\{ \frac{p_{wf}}{p} \right\} - 0.8 * \left\{ \frac{p_{wf}}{p} \right\}^2 \right) \quad (2)$$

q_{max} is the absolute open flow (AOF). Q_{max} is usually estimated using the reservoir pressure parameter and productivity index defined above the bubble point pressure. Mathematically, AOF is expressed as;

$$q_{max} = \frac{J * p}{1.8} \quad (3)$$

Vogel IPR for partial two phase reservoir is given by;

$$Q = q_b + q_v * \left(1 - 0.2 * \left\{ \frac{p_{wf}}{p_b} \right\} - 0.8 * \left\{ \frac{p_{wf}}{p_b} \right\}^2 \right) \quad (4)$$

$$q_b = J * (p - p_b) \quad (5)$$

$$q_v = \frac{J * p_b}{1.8} \quad (6)$$

Where p_b is the bubble point pressure, q_v is the Vogel flow rate and p is the average reservoir pressure. A typical Vogel IPR curve is described in Figure 3.

Developing IPR Curves Using Test Points

It is obvious that, IPR curves can be constructed using reservoir parameters including formation permeability, fluid viscosity, drainage area, wellbore radius, and well skin factor. These parameters assist in the determination of productivity index using the IPR model. However, some of these parameters are not always readily available. Thus, measured values of production rate and flowing bottom-hole pressure, are frequently used for constructing IPR curves⁸. These field measured values are known as test points.

³ Vogel, J.V. (1968). Inflow Performance Relationships for Solution-Gas Drive Wells. J Pet Technol 20 (1): 83–92. SPE 1476-PA. <http://dx.doi.org/10.2118/1476-PA>.

⁴ Standing, M. B. (1971). Concerning the calculation of inflow performance of wells producing from solution gas drive reservoirs. J. Pet. Tech. 23:1141–1142.

⁵ Fetkovich, M. (1973). The isochronal testing of oil wells. *SPE Annual Technical Conference and Exhibition*. Las Vegas, Nevada.

⁶ Bandakhlia, H., & Aziz, K. (1989). Inflow Performance Relationship for Solution gas drive horizontal wells. *SPE Annual Technical Conference and Exhibition*. San Antonio, Texas.

⁷ Retnanto, A., and Economides, M. J. (1996). Performance of multiple horizontal well laterals in low-to-medium-permeability reservoirs. *SPE Res. Eng.* 11:73–77.

⁸ Ghalambor, A., Lyons, W. C. & Guo, B. (2007). *Petroleum Production Engineering*. Elsevier Science and Technology Books.

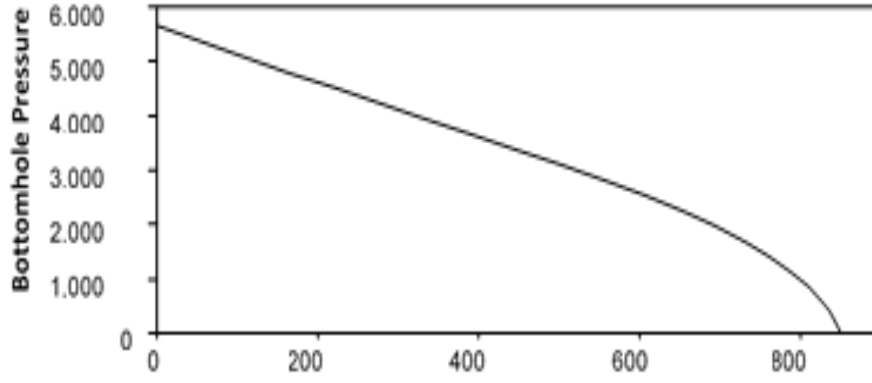


Figure 3: Typical Vogel IPR⁹

Drawing IPR curves using test points involves the direct calculation of the constants in the IPR models. For a single-phase (unsaturated oil) reservoir, the model constant J can be calculated as follows;

$$J = \frac{q_1}{P - P_{wf1}}$$

Where q_1 is the tested production rate at tested flowing bottom-hole pressure p_{wf1} .

If $P_{wf1} > P_b$ then

$$J = \frac{q_1}{P - P_{wf1}} \quad (7)$$

Else If $P_{wf1} < P_b$ then

$$J = \frac{q_1}{\left[(p - P_b) + \frac{P_b}{1.8} \left(1 - 0.2 \left(\frac{p_{wf}}{P_b} \right) - 0.8 \left(\frac{p_{wf}}{P_b} \right)^2 \right) \right]} \quad (8)$$

End if.

Eq. (7) & (8) can be used to determine the productivity index of a reservoir with known bottom hole flowing pressure and oil flowrate.

Tubing Performance Relationship (TPR) Model

Several TPR or VLP models have been developed for evaluating multiphase flow in vertical pipes. Brown¹⁰ presented a proper review of these models. TPR models for multiphase flow wells are in two categories

1. Homogeneous flow models and
2. Separated-flow models.

Homogeneous models do not consider the effects of liquid holdup i.e. there is a no-slip assumption. Homogeneous models are easily incorporated into computer models. The distinctive feature of

⁹ Ghalambor, A., Lyons, W. C. & Guo, B. (2007). *Petroleum Production Engineering*. Elsevier Science and Technology Books.

¹⁰ Brown, K. (1977). *The Technology of Artificial Lift Methods* (Vol. 1). Tulsa, OK: PentWell Books.

homogeneous models is attributed to the fact that they are mechanistic. These models can be used to handle either gas-oil water three-phase or gas-oil-water-sand four-phase systems

Nonetheless, separated-flow models are less ideal than homogeneous-flow models. They are usually expressed as empirical correlations. Unlike homogeneous flow models, separate flow models account for liquid hold-up. The main challenge with separated flow models is that, they are somewhat difficult to code in computer programs – owing to the fact that most correlations are presented in charts.

Homogeneous-Flow Models

Ever since Poettmann and Carpenter¹¹ developed a homogeneous-flow model analyzing TPR of multiphase wells, more of such models have also be developed. Poettmann–Carpenter’s model utilizes empirical double-phase friction factor for friction pressure loss calculations neglecting the effect of liquid viscosity. The effect of liquid viscosity was considered by later researchers like Dukler et al.¹² and Cicchitti¹³.

Poettmann and Carpenter Model

Poettmann and Carpenter presented a simplified gas-oil-water three phase flow model to calculate pressure losses in wellbores. The model was presented with an assumption that there is no slip of liquid phase. However, pressure losses are computed by estimating mixture density and friction factor. Neglecting acceleration due to gravity, the following equation can be used to calculate pressure drop in a vertical tubing¹⁴:

$$dp = \left(p + \frac{k}{p} \right) * \frac{dh}{144} \quad (8)$$

Where dh = increment per depth, ft

p = average mixture density, lb/ft³

$$k = \frac{f_{2f} q_o^2 M^2}{7.4137 * 10^{10} D^5} \quad (9)$$

Where f_{2f} is the fanning friction factor for a two phase flow, q_o is the oil flow rate measured in stb/day, D is the tubing internal diameter (ft) and M is the entire mass associated with 1stb of oil. Average mixture density is calculated as follows:

$$p = \frac{p_1 + p_2}{2}$$

Where

p_1 = Mixture density at top of tubing segment, lb/ft³

p_2 = Mixture density at bottom of segment, lb/ft³

¹¹ Poettmann, F., & Carpenter, P. (1952). Multiphase Flow of gas, oil and water through vertical flow strings with Application to Design lift Installations. *Drilling and Production Practice*, pp. 257-317.

¹² Dukler, A., Wicks, M., & Cleveland, R. (1964). Frictional pressure drop in two phase flow: A comparison of existing correlations for pressure loss and hold-up. *AIChE*, 38-42.

¹³ Cicchitti, A. (1960). Two Phase Cooling Experiments - Pressure drop, heat transfer and burnout measurements. *Energia Nucleare*, 7(6), 407.

¹⁴ Ghalambor, A., Lyons, W. C. & Guo, B. (2007). *Petroleum Production Engineering*. Elsevier Science and Technology Books.

The mass flow rate and volume flow rate can be used to compute mixture density at a given point on the tubing:

$$\rho = \frac{M}{V_m} \quad (10)$$

Where

$$M = 350.17(\gamma_o + WOR \gamma_w) + GOR \rho_{air} \gamma_g \quad (11)$$

$$V_m = 5.615(B_o + WOR B_w) + (GOR - R_s) \left(\frac{14.7}{p} \right) \left(\frac{T}{520} \right) \left(\frac{z}{1.0} \right) \quad (12)$$

Where γ_o = oil specific gravity, 1 for fresh water

WOR= producing water oil ratio, bbl/stb

GOR= producing gas-oil-ratio, scf/stb

ρ_{air} = Density of air, lbm/ft³

γ_g = Gas-specific gravity, 1 for air

V_m = volume of mixture associated with 1 stb of oil, ft³

B_o = formation volume factor of oil, rb/stb

B_w = formation volume factor of water, rb/bbl

R_s = solution gas–oil ratio, scf/stb

P = in situ pressure, psia

T = in situ temperature, °R

z = gas compressibility factor at p and T .

The double-phase friction factor f_{2F} can be computed from a graph as recommended by Poettmann and Carpenter¹⁵. For easy coding in computer programs, Guo and Ghalambor¹⁶ proposed the following correlation to be used in place of the chart:

$$f_{2F} = 10^{1.444 - 2.5 \log(Dpv)} \quad (13)$$

Where (Dpv) is the numerator of Reynolds number representing inertial force. It is computed as:

$$(Dpv) = \frac{1.4737 \times 10^{-5} M q_o}{D} \quad (14)$$

Guo–Ghalambor model

Guo–Ghalambor model is a mere advancement of the Poettmann and Carpenter's model. Guo and Ghalambor¹⁷ propounded a four-phase flow model which is similar to the gas-oil-water three-phase flow model presented by Poettmann and Carpenter. Guo–Ghalambor model also assumes no slip of

¹⁵ Poettmann, F., & Carpenter, P. (1952). Multiphase Flow of gas, oil and water through vertical flow strings with Application to Design lift Installations. *Drilling and Production Practice*, pp. 257-317.

¹⁶⁻¹⁷ Guo, B., & Ghalambor, A. (2002). *Gas Volume Requirements for Underbalanced Drilling Deviated Holes*. Tulsa: PennWell Corporation.

liquid phase. It is important however, to note that the Guo–Ghalambor model takes a closed (integrated) form, which makes it easy to use. The Guo-Ghalambor model can be expressed as follows:

$$144b(p_{wf} - p_{hf}) + \frac{1-2bM}{2} * \ln \left| \frac{(144p_{wf}+M)^2+N}{(144p_{hf}+M)^2+N} \right| - \frac{M+\frac{b}{c}N-bM^2}{\sqrt{N}} * \left[\tan^{-1} \left(\frac{144p_{wf}+M}{\sqrt{N}} \right) - \tan^{-1} \left(\frac{144p_{hf}+M}{\sqrt{N}} \right) \right] = a(\cos\theta + d^2e)L \quad (15)$$

Where the group parameters are defined as:

$$a = \frac{0.0765\gamma_g q_g + 350\gamma_o q_o + 350\gamma_w q_w + 62.4\gamma_s q_s}{4.07T_{av}q_g}$$

$$b = \frac{5.615q_o + 5.615q_w + q_s}{4.07T_{av}q_g}$$

$$c = 0.00678 \frac{T_{av}q_g}{A}$$

$$d = \frac{0.00166}{A} (5.615q_o + 5.615q_w + q_s)$$

$$e = \frac{f_m}{2gD_H}$$

$$M = \frac{cde}{(\cos\theta + d^2e)^2}$$

A = cross-sectional area of conduit, ft²

D_H = hydraulic diameter, ft

f_M = Darcy–Wiesbach friction factor (Moody factor)

g = gravitational acceleration, 32.17 ft/s²

L = conduit length, ft

p = pressure, psia

p_{hf} = wellhead flowing pressure, psia

q_g = gas production rate, scf/d

q_o = oil production rate, bbl/d

q_s = sand production rate, ft³/day

q_w = water production rate, bbl/d

T_{av} = average temperature, 8R

γ_g = specific gravity of gas, air = 1

γ_o = specific gravity of produced oil, freshwater = 1

γ_s = specific gravity of produced solid, fresh water = 1

γ_w = specific gravity of produced water, fresh water = 1

Darcy–Wiesbach friction factor (f_M) can be computed by using f_{2f} . F_{2f} can be obtained from Eq. (13). Darcy–Wiesbach friction factor is therefore expressed as:

$$f_M = 4f_{2f}.$$

Due to the complexity of Eq. (15), iterations will be required to provide the solution to the equation – i.e. Pwf. The mathematical model will be developed in the next section.

Separated-flow models

A few number of separated-flow models are available for TPR or VLP calculations. Some of them include; the Hagedorn and Brown method and the Duns and Ros correlation¹⁸.

Based on proper comparisons of these models, Ansari et al.¹⁹ presented the Hagedorn–Brown method with modifications for near-vertical flow. The modified Hagedorn–Brown method is an empirical correlation developed based on the work of Hagedorn and Brown²⁰. The Griffith correlation is usually used to compute bubble flow regime²¹. The modified Hagedorn–Brown correlation is expressed in oil field units as follows:

$$144 \frac{dp}{dz} = \rho + \frac{f m^2}{(7.413 \times 10^5 D^5) \rho} + \frac{\rho \left[\Delta \left(\frac{u_m^2}{2gc} \right) \right]}{\Delta z} \quad (16)$$

m = total mass flow rate, lbm/d

ρ = in situ average density, lbm/ft³

u_m = mixture velocity, ft/s

Notwithstanding, TPR model using Poettmann Carpenter's model is defined by:

$$p_{wf} = p_{wh} + \left(\rho + \frac{k}{\rho} \right) * \frac{L}{144} \quad (17)$$

METHODOLOGY

This section discusses the step by step methods employed in developing an analytic and computer model to achieve the objectives of this study.

About the Computer Model Developed (NODARES)

NODARES is a flexible software developed for performing Nodal Analysis considering only IPR and VLP or TPR. It can also be used to determine the stock tank oil originally in place for a solution gas drive reservoir with negligible water influx. These qualities of the NODARES software makes it peculiar and flexible – since it cuts across two sub-disciplines of Petroleum Engineering (i.e. both Production and Reservoir Engineering). However, the computer model is developed using Microsoft Visual Studio 2010 (VB.NET). The NODARES tool makes use of Poettmann Carpenter model and Rotimi and Wilfred model (a solution to Guo-Ghalambor model) to predict the bottomhole flowing pressure and optimum flowrate of a production system. The splash screen of NODARES software is as shown in Appendix A—1.

Brief Introduction about the Programming Language Used

¹⁸ Duns, H., & ROS, N. (1963). Vertical flow of gas and liquid mixtures in wells. *World Petroleum Congress*.

¹⁹ Ansari, A., Sylvester, Sarica, C., Shoham, O., & Brill, J. (1994, May). A Comprehensive Mechanistic Model for upward Two-Phase Flow in Wellbore. *SPE Production and Facilities*. AIME.

²⁰ Brown, K. (1977). *The Technology of Artificial Lift Methods* (Vol. 1). Tulsa, OK: PentWell Books.

²¹ Griffith, P., & Wallis, G. (1961). Two-Phase Slug Flow. *Trans, ASME*, 83, 307-315.

Microsoft Visual Basic 2010 is one of the most popular programming languages for developing Windows and web applications. In modern software development, the most important component is the “.NET Framework” - which is an essential component of every application. The Framework has an enormous collection of functions for just about any programming task²². These tasks include creating a new folder and calling the Create Directory method of the Directory class, and retrieving the files in a folder (by calling the GetFiles method of the same class). The Framework contains all the functionality of the operating system and makes it available to the developed application through the use of “methods”. Methods are very similar to functions, which extend the basic capabilities of a language. The Framework is a huge collection of such methods, structured in units according to their functionality and in a way that makes it fairly easy to locate the methods for the task at hand. The language and the Framework are the two “programming” components – that are absolutely necessary to build Windows and Web applications²³.

Analytic Model Developed for Nodal Analysis

Analytic models are mathematical models that have a closed form solution, i.e. the solution to the equations used to describe changes in a system that can be expressed in an exact mathematical function²⁴. Two analytic models are used in this project- one for nodal analysis and the other for reservoir volume estimation.

The previous section identified several models that can be used for conducting Nodal Analysis, amongst which, is the Guo-Ghalambor and Poettmann Carpenter model- the models used for this study. As observed in the second section of this study, Guo-Ghalambor model is a very complex model that can be used to determine the bottomhole flowing pressure of a producing or flowing well. This will in turn assist in the prediction of the optimum flowrate. Conventionally, performing nodal analysis requires the plotting of graphs so as to determine the point of intersection between IPR and VLP curves thus, providing the optimum flowrate in which the well should flow. However, with the evolution of empirical models, flowrate can be estimated without necessarily having to plot a graph. Nonetheless, these models are complex and can only be solved by iterations along with advanced computer programming. In a bid to use a complex model like Guo-Ghalambor, it is paramount that it is first simplified analytically –so that Newton Raphson iteration method can be used to provide a solution. The step by step simplification of the model is as shown below:

First recall the model as presented by Guo-Ghalambor²⁵

$$144b(p_{wf} - p_{hf}) + \frac{1-2bM}{2} * \ln \left| \frac{(144p_{wf}+M)^2+N}{(144p_{hf}+M)^2+N} \right| - \frac{M+\frac{b}{c}N-bM^2}{\sqrt{N}} * \left[\tan^{-1} \left(\frac{144p_{wf}+M}{\sqrt{N}} \right) - \tan^{-1} \left(\frac{144p_{hf}+M}{\sqrt{N}} \right) \right] = a(\cos\theta + d^2e)L \quad \text{i.e. eq.(15)}$$

22-23 Petroulous, E. (2002). Mastering Visual Basic.Net. San Francisco, London: SYBEX, Inc. Alameda. Retrieved from www.sybex.com

24 Oxford Dictionaries. (n.d.). Computer Model. Retrieved from https://en.oxforddictionaries.com/definition/computer_model

25 Ghalambor, A., Lyons, W. C. & Guo, B.(2007). Petroleum Production Engineering. Elsevier Science and Technology Books.

To calculate or predict the operating pressure, Guo-Ghalambor model is used. Recall this model from eq. (15)

$$144b(p_{wf} - p_{hf}) + \frac{1 - 2bM}{2} * \ln \left| \frac{(144p_{wf} + M)^2 + N}{(144p_{hf} + M)^2 + N} \right| - \frac{M + \frac{b}{c}N - bM^2}{\sqrt{N}} \\ \times \left[\tan^{-1} \left(\frac{144p_{wf} + M}{\sqrt{N}} \right) - \tan^{-1} \left(\frac{144p_{hf} + M}{\sqrt{N}} \right) \right] = a(\cos\theta + d^2e)L$$

Eq. (15) can also be solved using Newton Raphson's iteration method as follows;

$$f(p_{wf}) = 0 = 144b(p_{wf} - p_{hf}) + \frac{1 - 2bM}{2} * \ln \left| \frac{(144p_{wf} + M)^2 + N}{(144p_{hf} + M)^2 + N} \right| - \frac{M + \frac{b}{c}N - bM^2}{\sqrt{N}} \\ \times \left[\tan^{-1} \left(\frac{144p_{wf} + M}{\sqrt{N}} \right) - \tan^{-1} \left(\frac{144p_{hf} + M}{\sqrt{N}} \right) \right] - a(\cos\theta + d^2e)L$$

$$f(p_{wf}) = 144b(p_{wf} - p_{hf}) + \frac{1 - 2bM}{2} * \ln \left| \frac{(144p_{wf} + M)^2 + N}{(144p_{hf} + M)^2 + N} \right| - \frac{M + \frac{b}{c}N - bM^2}{\sqrt{N}} \times \\ \left[\tan^{-1} \left(\frac{144p_{wf} + M}{\sqrt{N}} \right) - \tan^{-1} \left(\frac{144p_{hf} + M}{\sqrt{N}} \right) \right] - a(\cos\theta + d^2e)L \quad (18)$$

For ease of solving, let;

$$Q = \frac{1 - 2bM}{2}$$

$$R = (144p_{hf} + M)^2 + N$$

$$T = \frac{M + \frac{b}{c}N - bM^2}{\sqrt{N}}$$

$$W = \tan^{-1} \left(\frac{144p_{hf} + M}{\sqrt{N}} \right)$$

$$Z = a(\cos\theta + d^2e)L$$

Thus, eq. (21) reduces to:

$$f(p_{wf}) = 144b(p_{wf} - p_{hf}) + Q * \ln \left| \frac{(144p_{wf} + M)^2 + N}{R} \right| - T \left[\tan^{-1} \left(\frac{144p_{wf} + M}{\sqrt{N}} \right) - W \right] - Z$$

(19)

$$f'(p_{wf}) = 144b + Q * \frac{d}{dp_{wf}} \left[\ln \left| \frac{(144p_{wf} + M)^2 + N}{R} \right| \right] - \frac{d}{dp_{wf}} \left[T \left(\tan^{-1} \left(\frac{144p_{wf} + M}{\sqrt{N}} \right) - W \right) \right] = 0$$

(20)

Simplifying each derivative,

$$\begin{aligned} \frac{d}{dp_{wf}} \left[Q \ln \left| \frac{(144p_{wf} + M)^2 + N}{R} \right| \right] &= \frac{R}{(144p_{wf} + M)^2 + N} \times \frac{2Q}{R} (144p_{wf} + M) \times 144 \\ &= \frac{288Q(144p_{wf} + M)}{(144p_{wf} + M)^2 + N} \end{aligned}$$

(21)

$$\text{Simplifying } \frac{d}{dp_{wf}} \left[T \left(\tan^{-1} \left(\frac{144p_{wf} + M}{\sqrt{N}} \right) - W \right) \right]$$

$$\text{Let } \alpha = T \left(\tan^{-1} \left(\frac{144p_{wf} + M}{\sqrt{N}} \right) \right)$$

$$\tan \left(\frac{\alpha}{T} \right) = \left(\frac{144p_{wf} + M}{\sqrt{N}} \right)$$

(22)

$$p_{wf} = \frac{\tan \left(\frac{\alpha}{T} \right) \sqrt{N} - M}{144}$$

$$\frac{dp_{wf}}{d\alpha} = \frac{1}{144} \times \frac{d}{d\alpha} \left(\tan \left(\frac{\alpha}{T} \right) \sqrt{N} - M \right)$$

$$\frac{dp_{wf}}{d\alpha} = \frac{\sqrt{N}}{144} \times \frac{d}{d\alpha} \left(\tan \left(\frac{\alpha}{T} \right) \right)$$

$$\frac{dp_{wf}}{d\alpha} = \frac{\sqrt{N}}{144} \times \frac{1}{T} \sec^2 \left(\frac{\alpha}{T} \right)$$

$$\text{But } \sec^2 \left(\frac{\alpha}{T} \right) = 1 + \tan^2 \left(\frac{\alpha}{T} \right)$$

Thus,

$$\frac{dp_{wf}}{d\alpha} = \frac{\sqrt{N}}{144} \times \frac{1}{T} \left(1 + \tan^2 \left(\frac{\alpha}{T} \right) \right) \quad (23)$$

Put eq. (22) into eq. (23)

$$\frac{dp_{wf}}{d\alpha} = \frac{\sqrt{N}}{144} \times \frac{1}{T} \left(1 + \left(\frac{(144p_{wf} + M)}{\sqrt{N}} \right)^2 \right)$$

$$\frac{dp_{wf}}{d\alpha} = \frac{\sqrt{N}}{144} \times \frac{1}{T} \left(1 + \left(\frac{144p_{wf} + M}{\sqrt{N}} \right)^2 \right)$$

$$\frac{dp_{wf}}{d\alpha} = \frac{\sqrt{N}}{144} \times \frac{1}{T} \left(\frac{N + (144p_{wf} + M)^2}{N} \right)$$

$$\frac{dp_{wf}}{d\alpha} = \frac{\sqrt{N}}{144} \times \frac{1}{T} \left(\frac{N + (144p_{wf} + M)^2}{(\sqrt{N})^2} \right)$$

$$\frac{dp_{wf}}{d\alpha} = \left(\frac{N + (144p_{wf} + M)^2}{144\sqrt{NT^2}} \right)$$

$$\frac{d\alpha}{dp_{wf}} = \left(\frac{dp_{wf}}{d\alpha} \right)^{-1}$$

$$\frac{d\alpha}{dp_{wf}} = \left(\frac{N + (144p_{wf} + M)^2}{144\sqrt{NT^2}} \right)^{-1}$$

$$\frac{d\alpha}{dp_{wf}} = \frac{144\sqrt{NT^2}}{N + (144p_{wf} + M)^2} \quad (24)$$

Putting the solutions of the derivatives (eq. (21) & eq. (24)) into eq. (20)

$$f'(p_{wf}) = 144b + \frac{288Q(144p_{wf} + M)}{(144p_{wf} + M)^2 + N} - \frac{144\sqrt{NT^2}}{N + (144p_{wf} + M)^2} \quad (25)$$

Applying Newton Raphson iteration algorithm,

$$p_{wfi+1} = p_{wfi} + \frac{f(p_{wf})}{f'(p_{wf})} \quad (26)$$

Eq. (26) is incorporated into a computer model and is looped until the value of p_{wf} is generated with an error less than 0.01%. Once p_{wf} is known, the optimum flowrate can be estimated by substitution of the value of p_{wf} into eq. (1).

Eq. (25) is therefore, Rotimi and Wilfred Model - which provides a solution to Guo-Ghalobor model by the application of the Newton Raphson algorithm.

Anyways, for the purpose of visual representation, Poettmann Carpenter Model is used. IPR model for single phase (straight line) IPR is given by:

$$q = J * (P - p_{wf}) \quad (a)$$

TPR model is defined by:

$$p_{wf} = p_{wh} + \left(\rho + \frac{k}{\rho} \right) * \frac{L}{144} \quad (b)$$

Where the value of k is as defined as:

$$k = \frac{f_{2f} q_o^2 M^2}{7.4137 * 10^{10} D^5} \quad \text{i.e. eq. (9)}$$

And M is given by:

$$M = 350.17(\gamma_o + WOR \gamma_w) + GOR p_{air} \gamma_g \quad \text{i.e. eq. (11)}$$

Where $p_{air} = 0.074887 \text{ lbm/cuft}$

Computer Model

A computer model refers to a representation of a system created on a computer, to assist calculations and make predictions. It also refers to a series of codes written in a computer to perform specific functions which are usually tasking or too complex to be done analytically. The computer model to be developed in this project will perform both Nodal Analysis and reservoir volume estimation. The software is named after its function. It is called NODARES. The flowchart for developing the NODARES software is presented in Appendix A-3.

In a bid to develop a software (computer model) to perform Nodal Analysis, Vb.net 2010 is used. The flowchart for performing Nodal Analysis is as shown in Appendix A—2 and A--3

Step By Step Approach In Predicting the Bottomhole Flowing Pressure and Optimum Flowrate Using NODARES

Nodal Analysis can be performed using either Rotimi and Wilfred model or Poettmann Carpenter model. The former model, however, can perform predictions without displaying charts while the later uses charts to show the relationship between IPR and VLP.

1. Launch the NODARES software and select the appropriate model from the “Production tab” on the menu strip. If Rotimi and Wilfred model is selected, click on “model” from the tree view that appears afterwards.
2. Fill the VLP and IPR data using the input data provided in Table 1, and click on calculate to view result.
3. However, if Poettmann Carpenter model is selected, a new form pops out requesting for input data. Enter the input data using the data provided in Table 2 and click on the calculate button to display pressure and flowrate variables.
4. Click on “plot graph” to view the IPR versus VLP plot.

The figures shown below i.e., Figure 4 and Figure 5 illustrate the input data entry for Rotimi and Wilfred, and Poettmann Carpenter model respectively.

The screenshot shows the NODARES software interface. The 'Production' tab is selected in the menu bar. The 'VLP DATA' section contains the following inputs:

Parameter	Value	Unit
Tubing shoe depth	7000	Ft
Inclination angle	20	degrees
Tubing internal diameter	1.995	inches
Gas rate	1	MMscf/day
Gas specific gravity	0.7	-
Oil production rate	1000	bbl/day
Oil specific gravity	0.85	-
Water rate	300	bbl/day
Water specific gravity	1.05	-
Tubing head temperature	100	deg. F
Bottomhole temperature	224	deg. F
Wellhead pressure	300	psia
Darcy Weisbach friction factor	0.0415505	-
Solid production rate	1	cuft/day
Solid specific gravity	2.65	-

The 'IPR DATA' section contains the following inputs:

Parameter	Value	Unit
Reservoir pressure	3000	psia
P.I above bubble point	1.0	bbl/day-psi

A blue 'CALCULATE' button is located above the 'IPR DATA' section. On the right, a 'Model' tree view is visible.

Figure 4: Data entry for Rotimi and Wilfred Model

POETTMMAN CARPENTER MODEL

IPR TYPE: Straight Line

INPUT DATA

Tubing shoe depth	5000	Ft
Average mixture density	20.46	lb/cuft
Wellhead pressure	500	psia
Reservoir Pressure	3000	psia
Gas Liquid Ratio	1000	scf/stb
Fanning friction factor	0.0084	-
Tubing ID	1.66	in
Productivity Index	1.0	stb/day-psi
Gas Specific gravity	0.65	-
Oil Specific gravity	0.85	-
Liquid Rate	2000	stb/day
Water cut	25	%

RESULT

IPR	Flowrate	TPR

Buttons: PLOT GRAPH, CALCULATE

Figure 5: Data entry for Poettmann Carpenter model

Table 1: Input data for Rotimi and Wilfred Model²⁶

Parameters	Well-P55	Well-P71
Tubing shoe depth	7000 ft	5000ft
Inclination angle	20°	20°
Tubing internal diameter	1.995 in	1.995 in
Gas rate	1MMscf/day	0.83MMscf/day
Gas specific gravity	0.7	0.7
Oil production rate	1000bbl/day	1000bbl/day
Oil specific gravity	0.85	0.85
Water rate	300bbl	350bbl
Water specific gravity	1.05	1.05
Tubing head temperature	100°F	100°F
Bottomhole Temperature	224°F	160°F
Wellhead pressure	500psi	300psi
Darcy Weisbach friction factor.	0.415505	0.415505
Solid rate	1 cuft	1cuft
Solid specific gravity	2.65	2.65
Reservoir pressure	3000psi	3000psi
PI above bubble point	1.0bbl/day-psi	0.99bbl/day-psi

²⁶ Ghalambor, A., Lyons, W. C. & Guo, B.(2007). *Petroleum Production Engineering*. Elsevier Science and Technology Books.

Table 2: Input data for Poettmann Carpenter model²⁷.

Parameters	Well-P79
Tubing shoe depth	5000ft
Average mixture density	20.46lbm/cuft
Tubing internal diameter	1.66in
Water cut	25%
Gas Liquid Ratio	1000scf/stb
Liquid production rate	2000stb/day
Oil specific gravity	0.85
Gas specific gravity	0.65
Wellhead pressure	500psi
Fanning friction factor.	0.0084
Reservoir pressure	3000psi

RESULTS AND DISCUSSION

This section disc analysis of the results obtained from the NODARES tool for a range of data points and also presents its competence.

Results

The results obtained from using the NODARES tool are as shown in the figures below.

²⁷ Ghalambor, A., Lyons, W. C. & Guo, B.(2007). *Petroleum Production Engineering*. Elsevier Science and Technology Books.

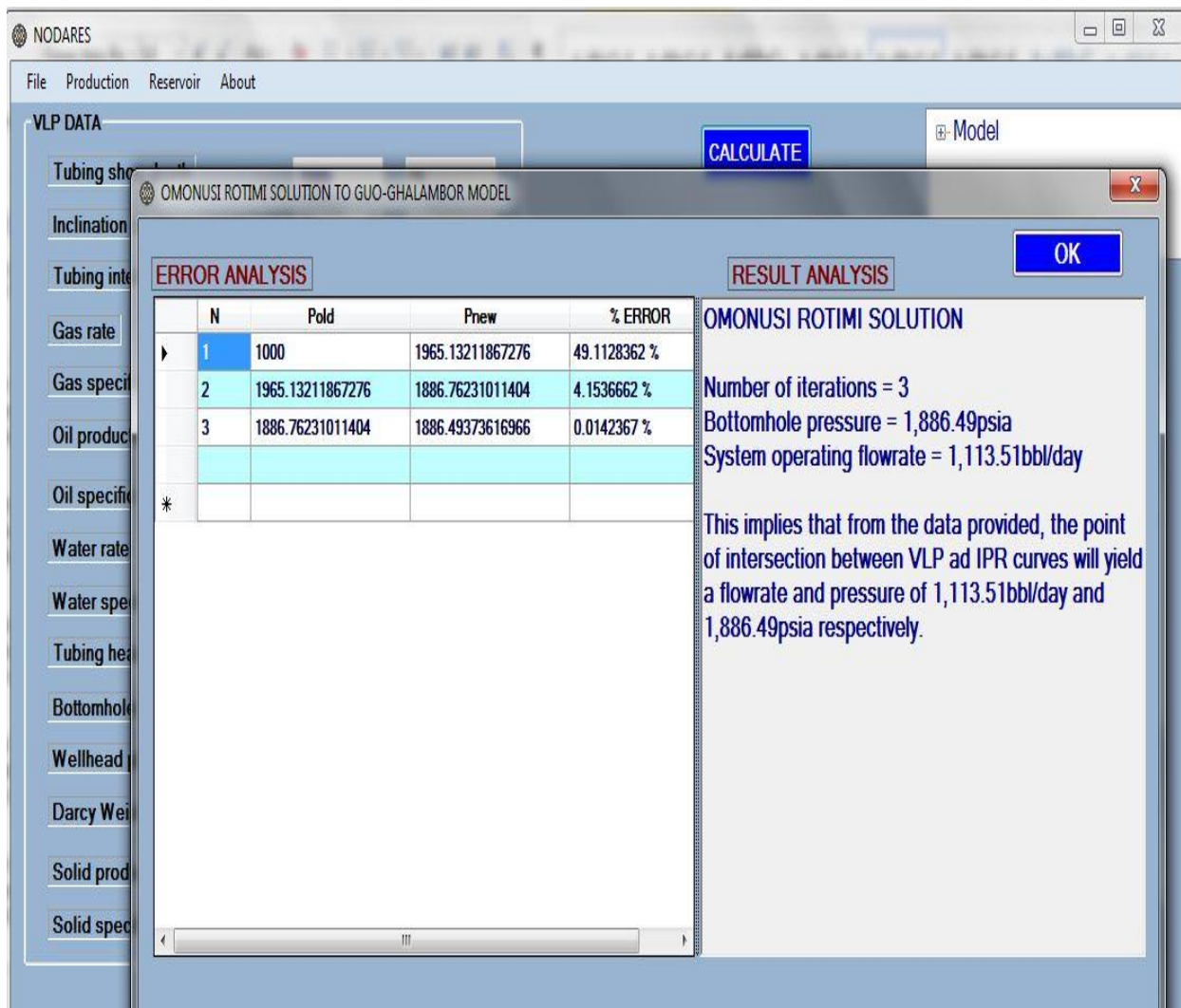


Figure 6: Result analysis for well-P55 using Rotimi and Wilfred Model

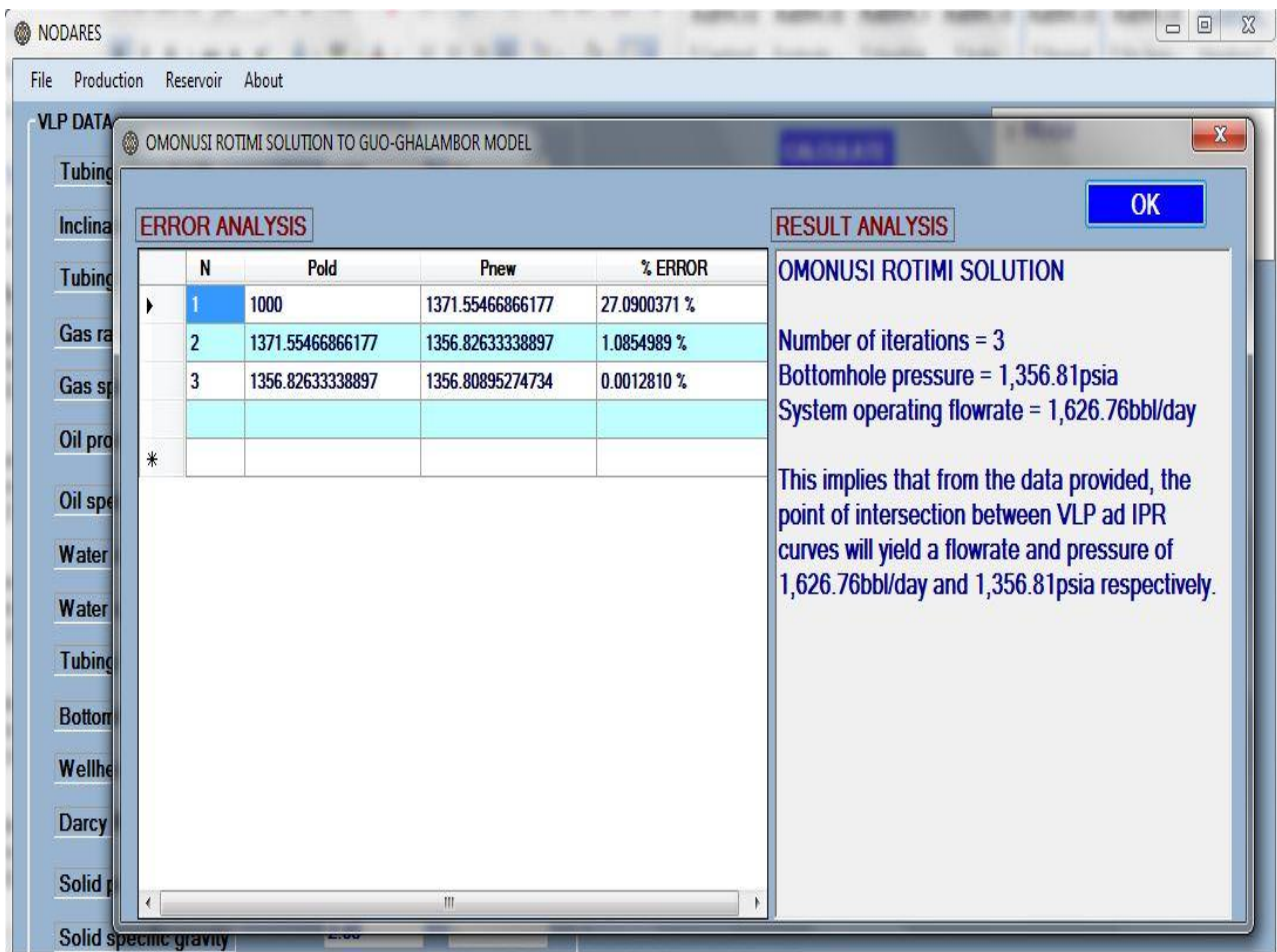


Figure 7: Result analysis for well-P71 using Rotimi and Wilfred Model

EXCEL SOLVER SOLUTION			
INPUT DATA		OUTPUT DATA	
OIL FLOWRATE	1000 psia	Area	3.125904325 sq. feet
OIL SG	0.85	a	0.000138736
WATER FLOWRATE	300 bbl/day	b	2.88382E-06
WATER SG	1.05	c	1349100.792
AVERAGE TEMPERATURE	622 deg. Rankine	d	3.876903686
Tubing ID	1.995 inches	e	0.003884486
GAS FLOWRATE	1000000 Scf/day	M	26158.22097
GAS SG	0.7	N	6669284618
INCLINATION ANGLE	20 degrees	QT	0.424564458
TBG LENGTH	7000 ft	R	16304320963
SOLID RATE	1 lbm/cuft	T	0.296146107
SOLID SG	2.65	W	50.24021691
WELLHEAD PRESSURE	500 psia	Z	0.969284345
FRICTION FACTOR	0.0415505	f(pwf)	-4.61187736
		pwf	1000 psia
		Sum of squares	21.26941281

Figure 8: Result analysis for well-P55 using EXCEL SOLVER

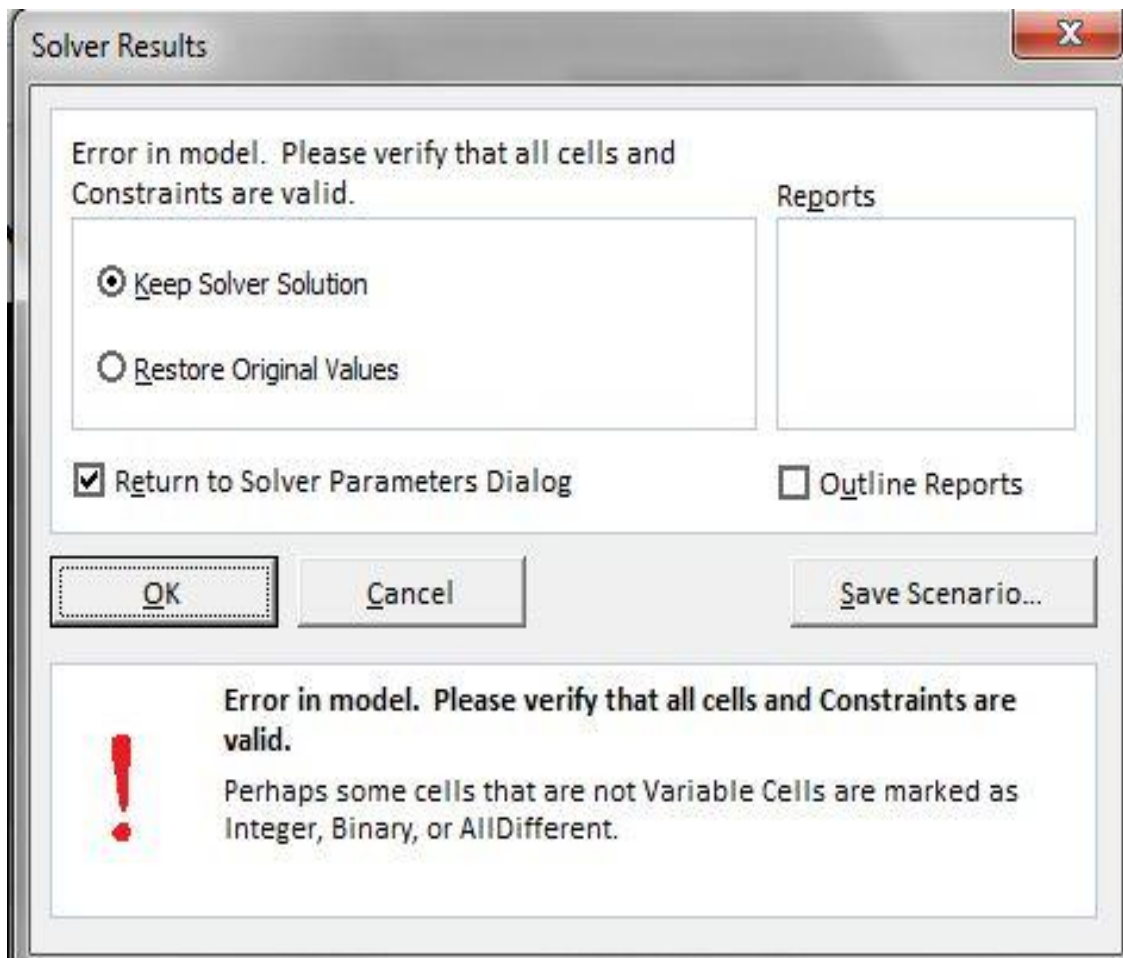


Figure 9: Excel Solver Report

Comparative Analysis between NODARES and Excel Solver

The results obtained using Excel Solver is as shown in figure 8 above. However, it was observed that excel solver could not generate a feasible solution to the complex Guo-Ghalambor model. The sum of squares of the error could not be zeroed; consequently, the initial guess of 1000 remained the same for the bottomhole flowing pressure. The report by excel solver is as shown in figure 9. It is therefore evident, that Excel Solver is not suitable for providing a solution to Guo-Ghalambor model. Moreover, NODARES software through the use of the Rotimi and Wilfred model, is most suitable to provide a solution to Guo-Ghalambor model.

POETTMANN CARPENTER MODEL

IPR TYPE: Straight Line

PLOT GRAPH CALCULATE

INPUT DATA

Tubing shoe depth	5000	Ft
Average mixture density	20.46	lb/cuft
Wellhead pressure	500	psia
Reservoir Pressure	3000	psia
Gas Liquid Ratio	1000	scf/stb
Fanning friction factor	0.0084	-
Tubing ID	1.66	in
Productivity Index	1.0	stb/day-psi
Gas Specific gravity	0.65	-
Oil Specific gravity	0.85	-
Liquid Rate	2000	stb/day
Water cut	25	%

RESULT

IPR	Flowrate	TPR
3000	0	1210.416666666
2500	500	1433.735510808
2000	1000	2103.692043232
1500	1500	3220.286263939
1000	2000	4783.518172929
500	2500	6793.387770202
0	3000	9249.895055758

Figure 10: Data entry and display of calculated variables for Well P-79 Using Straightline IPR

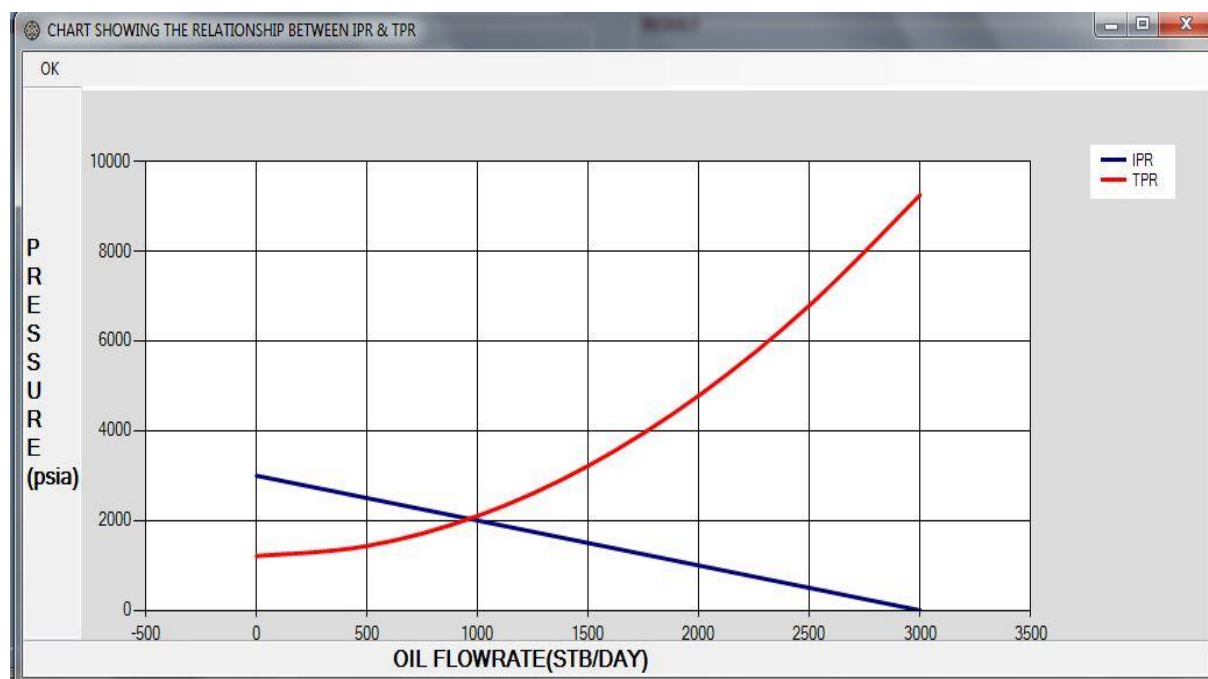


Figure 11: Nodal Analysis (Straight line IPR) plot for Well P-79

POETTMMANN CARPENTER MODEL

IPR TYPE **Vogel**

INPUT DATA

Tubing shoe depth	5000	Ft
Average mixture density	20.46	lb/cuft
Wellhead pressure	500	psia
Reservoir Pressure	3000	psia
Gas Liquid Ratio	1000	scf/stb
Fanning friction factor	0.0084	-
Tubing ID	1.66	in
Productivity Index	1.0	stb/day-psi
Gas Specific gravity	0.65	-
Oil Specific gravity	0.85	-
Liquid Rate	2000	stb/day
Water cut	25	%

RESULT

IPR	Flowrate	TPR
3000	0	1210.416666666
2500	466.666666666	1404.952193118
2000	850	1855.808126235
1500	1166.666666666	2426.263706992
1000	1400	2961.236404735
500	1566.666666666	3402.911452037
0	1666.666666666	3691.737157127

PLOT GRAPH CALCULATE

Figure 12: Data entry and display of calculated variables for Well P-79 Using Vogel IPR

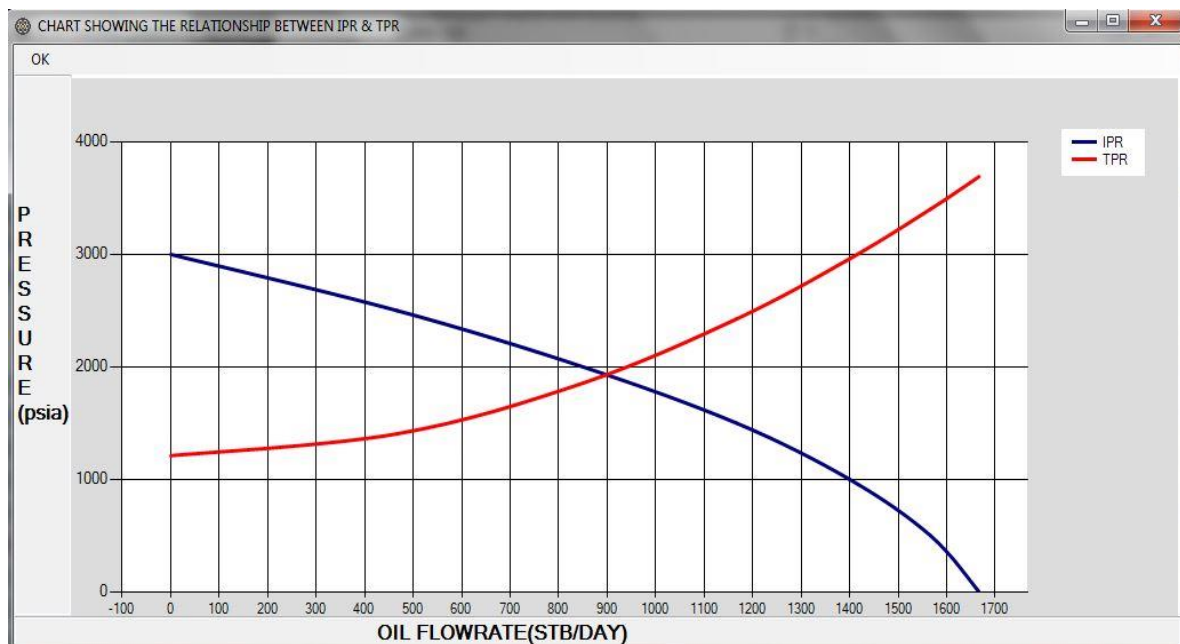


Figure 13: Nodal Analysis (Vogel IPR) plot for Well P-79

Discussion of results obtained using Rotimi and Wilfred model

The results obtained for well P-55 and Well P-71 are as shown in figure 6 and figure 7 respectively. After performing three iterations, a bottomhole flowing pressure of 1886.49psia was obtained for Well P-55. The optimum flowrate was also estimated as 1,113.51bbl/day. Similarly, the bottomhole flowing pressure and operating flowrate for Well-P71 was estimated as 1,356.81psia and 1,626.76bbl/day respectively. The results obtained from the two sets of data points has shown that, convergence is fast using the Rotimi and Wilfred Model.

Discussion of results obtained using Poettmann Carpenter's model

Nodal analysis plots for Well P-79 are as shown in Figure 11 and Figure13. First, a straight line IPR was used in analyzing well- P79. Thereafter, a Vogel IPR was also used. The data entries and display of estimated variables for Well P-79 are illustrated in Figure 10 and Figure 12.

The nodal analysis plot for well P-79 using straight line IPR shows that, the point of intersection between the VLP and IPR gave a bottomhole flowing pressure of 2000psia and an optimum oil flowrate of 970stb/day. Similarly, using Vogel IPR, the bottomhole flowing pressure and optimum flowrate for well P79 gave 1920psi and 900stb/day respectively. It should be noted however that, this model was incorporated into the NODARES tool for visual verification.

CONCLUSION

The Rotimi and Wilfred mathematical model was incorporated into a computer model for the sake of accurately predicting the bottomhole pressure and optimum flowrate. However, Poettmann Carpenter's model was also incorporated into the computer model – so as to enable the display of Nodal Analysis charts for visual verification. The results obtained from performing nodal analysis on Well P-55 and Well P-71 returned a value for the bottom hole flowing pressure having performed barely three iterations. This shows that convergence is fast using the Rotimi and Wilfred model. However, the limitations of this study are stated as follows:

- i. It performs Nodal Analysis for oil wells without considering surface facilities,
- ii. It takes into consideration, Inflow Performance Relationship (IPR) and Vertical Lift Performance (VLP) only,
- iii. The Vertical Lift Performance (VLP) curve is generated using homogenous flow models neglecting liquid hold ups in the tubing.

Nomenclature

ρ_1 = Mixture density at top of tubing segment, lb/ft³

ρ_2 = Mixture density at bottom of segment, lb/ft³

ρ_{air} = Density of air, lbm/ft³

γ_g = specific gravity of gas, air = 1

γ_o = specific gravity of produced oil, freshwater = 1

γ_s = specific gravity of produced solid, fresh water = 1

γ_w = specific gravity of produced water, fresh water = 1

A = cross-sectional area of conduit, ft²

D_H = hydraulic diameter, ft

f_M = Darcy–Wiesbach friction factor (Moody factor)

g = gravitational acceleration, 32.17 ft/s²

GOR= producing gas-oil-ratio, scf/stb

p_{hf} = wellhead flowing pressure, psia

q_g = gas production rate, scf/d

q_o = oil production rate, bbl/d

q_s = sand production rate, ft³/day

q_w = water production rate, bbl/d

T_{av} = average temperature, °R

u_m = mixture velocity, ft/s

WOR= producing water oil ratio, bbl/stb

z = gas compressibility factor at p and T.

ρ = in situ average density, lbm/ft³

Acknowledgement

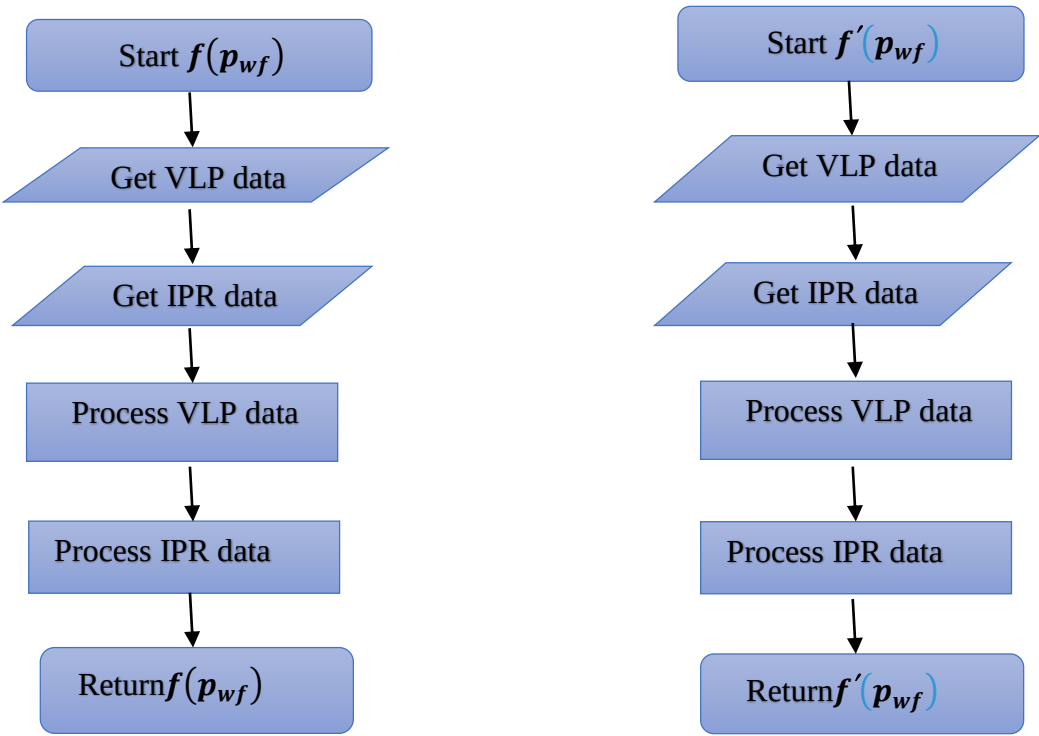
We express our profound gratitude to Petroleum Technology Development Fund (PTDF), Abuja for providing this wonderful platform to publish this work.

Appendix

A—1: SPLASHSCREEN (Computer Model Splashscreen)



A—2: Function flowchart (Nodal analysis function flowchart)



A—3: Computer Model Flowchart; Nodal Analysis flowchart for the computer model (NODARES)

